The Speed of Sound and Elastic Constants

Introductory Material

In this experiment, you will use a time-of-flight method to measure the speed of sound in liquids and solids. From the speed of sound, and the density, you will then find the compressional elastic constants of the materials.

The technique for doing this is called the time-of-flight method. It involves sending a pulse of sound from a transmitter transducer to a receiver transducer at a known distance away. The distance, Δx, is measured with a calipers and the travel time, Δt, is measured on an oscilloscope. The speed of sound is calculate from the slope, $\Delta x/\Delta t$.

The speed of sound in terms of material properties, V_s , in a material is given by

$$V_s = \sqrt{(C_C/\rho)}$$

where C_C is the compressional elastic constant of the material and ρ is the density. Once the speed of sound has been measured, this equation can be used to calculate the elastic constant as follows

$$C_C = \rho V_s^2$$

A liquid supports only one type of sound wave, a compressional wave, also called a longitudinal wave, which is shown schematically in the upper part of figure 1-1.

In a solid there are two kinds of sound waves, compressional and shear, also called transverse. The particle motion of shear waves is shown in the lower part of figure 1-1. In a solid the compressional wave is also called a "P wave", and the shear wave is also called an "S wave".

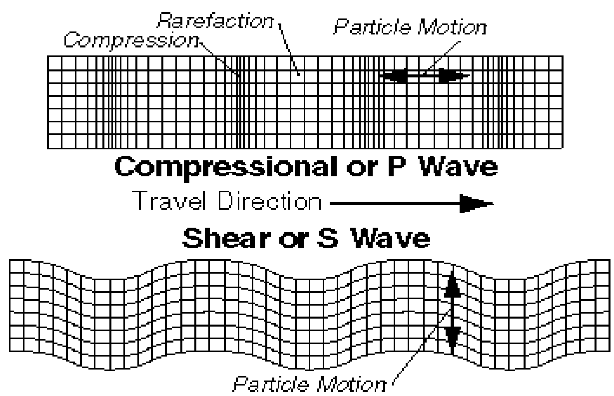

Figure 1-1. Compressional and shear waves.

Below is a table of Elastic Constants, Density, and the Speeds of the two type of sound waves, compressional and shear. All of this data is at 20° C and atmospheric pressure. The values in this table may not be exactly the same as those for your samples. The exact values depend on chemical content and mechanical treatment of the material.

	Compressional Elastic Constant (10^9 Newt/m^2)	Density (kg/m^3)	Speed of Compressional Wave (m/sec)	Speed of Shear Wave (m/sec)
Aluminum	111	2700	6420	3040
Copper	196	8930	4690	2230
Stainless Steel	265	7890	5790	3235
Diamond	1075	3520		11650
Polycarbonate	6.8	1180	2400	1070
Water	2.24	1000	1497	*
Air	0.000141	0.0012	343	*

* Shear waves do not exist in fluids.

There are several things to notice in the table. First, the shear waves are always slower than the compressional waves. Second, because of the formula for sound speed, the fastest sound wave will be in a material which is rigid (hard), that is, it has a large elastic constant, and low density. The fastest sound wave is in diamond, hard and light. In air sound travels very slowly because the elastic constant of air is very low.

Prelab Questions

1. What is the speed of compressional waves in diamond?
2. You have a knife made of a material with an elastic constant of 10^3 Newt/m^2. Would you be able to cut butter with it? Explain.
3. Derive equation 1-1.

Equipment List

Oscilloscope
"Sound Calipers" with mounting rod
Pulser Box
Water tray or Petri dish
Samples of plastic and metals
BNC patch cable (2)
Ring stand and cross clamp
Calipers
Scale

Experimental Procedure

Part I. Speed of sound in water:

The speed of sound in water will be measured with a “Sound Calipers”. This is a pair of ultrasonic transducers mounted on an electronic calipers. One of the transducers is connected to a pulser box, which applies very short pulses to the transducer. The transducer then transmits very short ultrasonic pulses. The second transducer receives the pulses and both pulses are displayed on an oscilloscope. The pulses are repeated every millisecond and the oscilloscope is set to sweep automatically every time a pulse occurs.

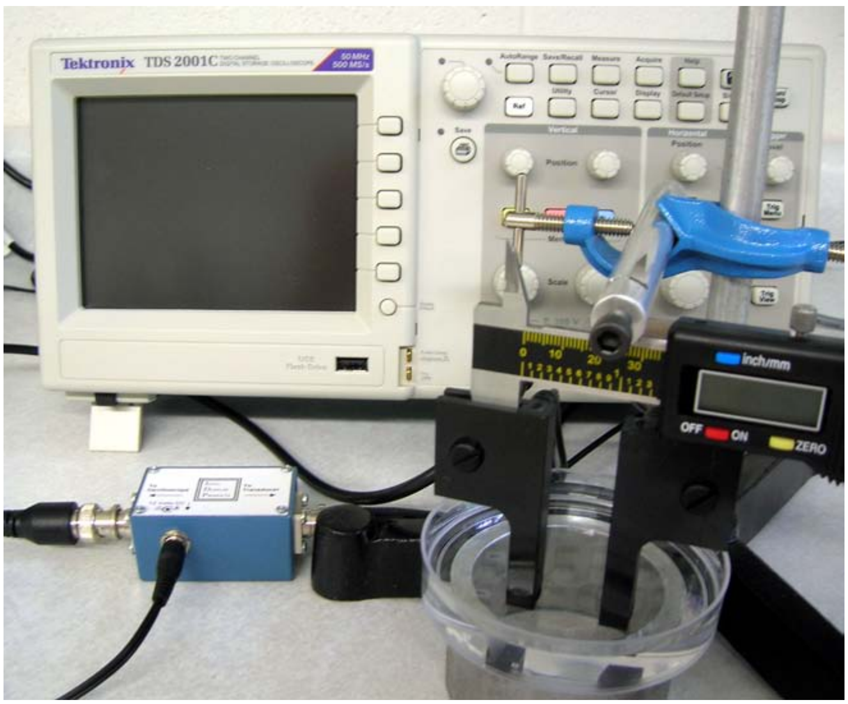

Figure 1-3. Setup for measuring the speed of sound in water.

- Place the Sound Heads in the water tray as shown in figure 1–4. The tips of the Sound Heads should be 8 to 10 mm below the water line.

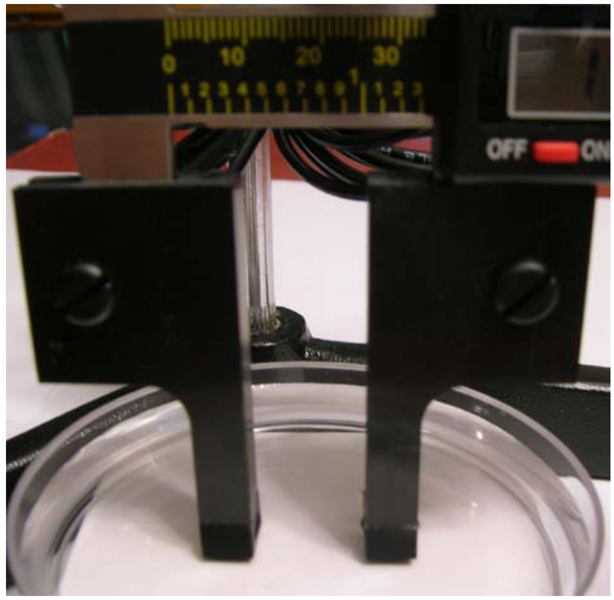

Figure 1-4. Close up of Sound Heads in water.

- Adjust the caliper separation, Δx, to approximately 1 cm.

- Check that two waveforms appear on the oscilloscope as shown in figure 1-5. Adjust the horizontal scale and the horizontal position so that the two pulses are at least a half a screen apart.

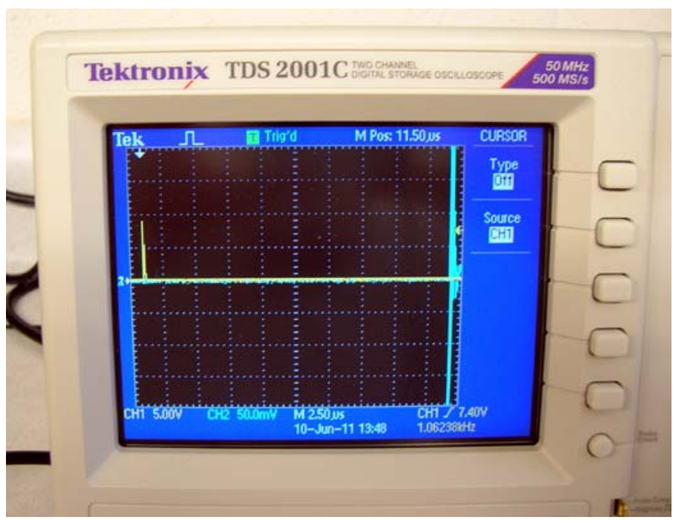

Figure 1-5. Oscilloscope with probes in place.

- The nominal oscilloscope settings will be as follows

CH1	2 V/div
CH2	100 mV/div
SEC/DIV	5 μsec
Trigger source	CH1
Trigger threshold	in the range of 1V to 5V
Trigger mode:	NORMAL

- Use the time cursors, placed at the leading edges of the pulses, to precisely measure the time difference, Δt.

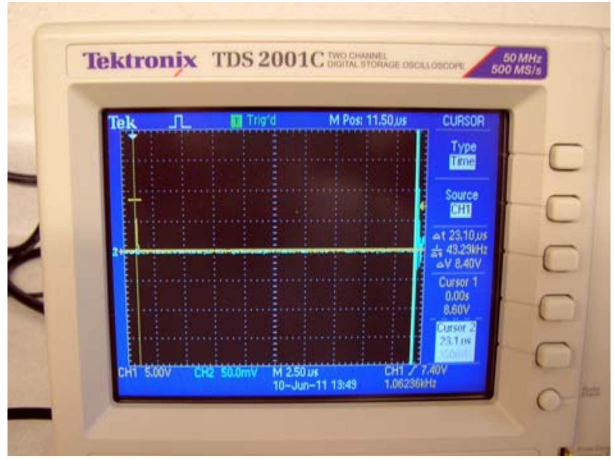

Figure 1-6. Cursor positions

- Record the separation, Δx, from the reading on the sound calipers, and the time delay, Δt, from the oscilloscope.
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- Repeat for three separations of approximately Δx = 2 cm, 3 cm, and 4 cm.

- Plot Δx on the vertical axis versus Δt on the horizontal axis using Graphical Analysis or other plotting software. The origin should appear on the plot.

- Perform a fit of the data with the equation y = mx + b. The speed of sound will be the slope m. Compare this to the accepted value of the speed of sound in water.

Part II: Speed of sound in a solid immersed in water:

The setup for this part is the same as the previous part, shown in figure 1-3.

The measurement will yield the speed of compressional sound waves (P waves) in a sample of solid material. This method has the advantage that it does not attach transducers to the solid; and, since the wave length is very small compared to any dimension of the sample, the resulting speed of sound will depend on the bulk value of the elastic constant rather than Young's modulus.

With a solid sample placed in the water tray between the transducers, as shown in figure 1-7, the time-of-flight, t_{tot},, will be

$$t_{tot} = t_{in\ water} + t_{in\ sample} = L_{water}/V_w + L_{sample}/V_s$$

where L_{water} is the length sound travels in water, L_{sample} is the length sound travels in the sample, V_w and V_s are the sound speeds in water and the sample respectively.

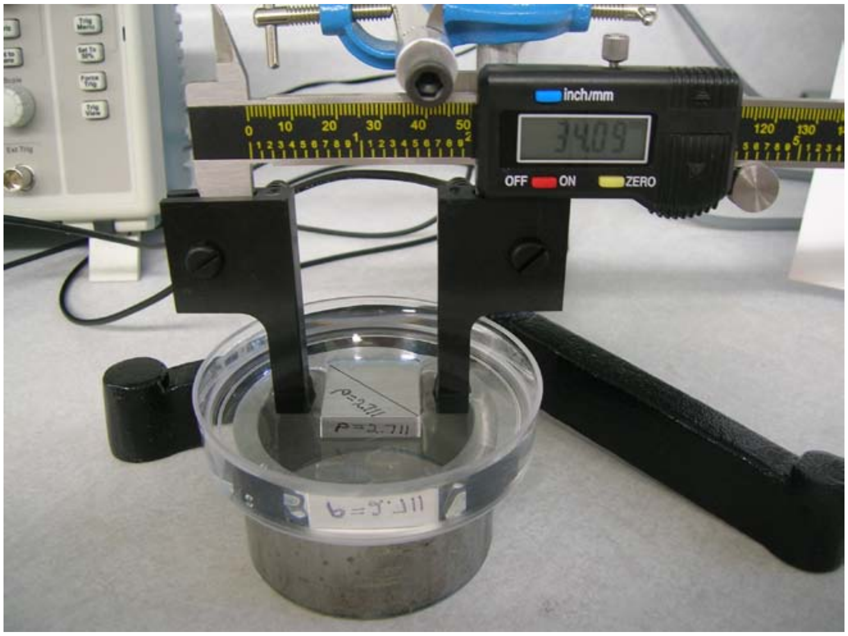

Figure 1-7. Solid sample placed between the transducers.

Without the sample,

$$t = (L_{water} + L_{sample})/V_w$$

and the difference is $\Delta t = t - t_{tot}$, which is the Δt, measured on the oscilloscope, of the arrival time with and without the sample between the transducers. This Δt is not the same as that measured in the previous part. The speed of sound in the solid is solved from the two equations above. The result is

$$V_s = \frac{1}{\frac{1}{V_w} - \frac{\Delta t}{L_{sample}}} \qquad \text{Eq. 1-1}$$

You will measure Δt for multiple sample thicknesses L_{sample} .

Measure L_{sample} , the thickness of your samples of metal or plastic, using a calipers. Record these values on your data sheet and calculate the speed of sound, V_s , for each thickness. Average the several measurements and compare to the accepted speed of sound in the material.

Repeat the measurements for another type of solid.

Part III. Elastic constants

Measure the density of the samples as follows:

- Weigh the sample on the scale.
- Place a container of water on the scale and record its weight.
- Hang the sample from a thread and lower it into the water.
- When it is completely submerged, read the scale.
- The difference in weight before and after the sample is submerged indirectly gives you the volume, and the density is just the sample weight divided by the volume.

For each of the results in the first two parts, calculate the elastic constant for the material involved. Compare these to the accepted values.

Questions

1. Observe the signal of the receiver transducer after the initial pulse arrives. What could be the source of some of the pulses that follow the initial pulse?

2. What is the least accurate measurement of all those you have made in this experiment? That is, what limits the final accuracy of your value for elastic constant? Explain why.

3. There are two sources of signal loss when you place a solid sample between the transducers. What are they?