Experiments in Acoustical Refraction and Diffraction

Table of Contents

Electromagnetic Wave Scattering at an Interface

Acoustical refraction is somewhat more complicated than EM refraction. Plane EM waves, at a planar interface between two different media, results in a reflected wave and a refracted wave. The procedure for calculating amplitudes and angles of the resultant waves is as follows:

1. Choose appropriate solutions to the wave equation.
2. Find parameters in these waves which satisfy the EM boundary conditions.

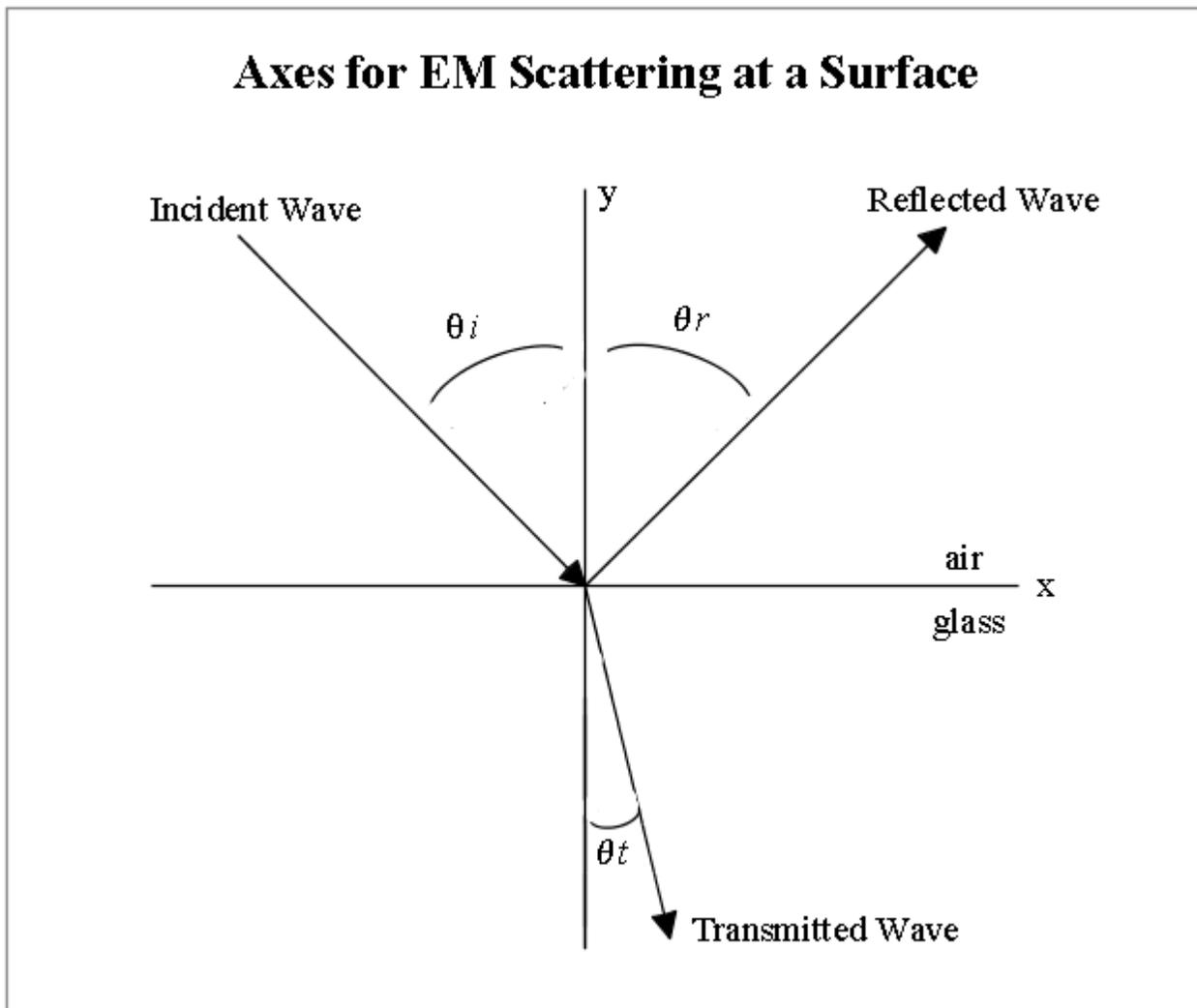The simplest arrangement is shown in figure 1.

Figure 1.

The three waves are simple z-polarized plane waves with incident E field,

$$\mathbf{E} = \mathbf{k}E_z e^{i(\omega - k_x x + k_y y)}$$

and incident H field,

$$\mathbf{H} = (\mathbf{i}H_x + \mathbf{j}H_y)e^{i(\omega - k_x x + k_y y)}$$

     The boundary conditions at y=0 are that E fields and H fields parallel to the interface must be continuous across the boundary. In other words, $E_z(0) = E_z'(0)$, $\quad E_x(0) = E_x'(0)$, $\quad H_z(0) = H_z'(0)$, and $H_x(0) = H_x'(0)$, where the primes indicate the values below the interface and unprimed fields are above the interface.[1] In this case there is no E_x or H_z on either side, so there are just two equations. Since the crests and troughs of the waves must line up at the interface, the value for k_x must be the same for all three waves. This is **Snell's law**, which can be used to calculate the propagation directions of the reflected and transmitted waves. The amplitudes of the reflected and transmitted waves can be calculated from the two boundary condition equations. Figure 2 shows results of the reflected and transmitted amplitudes and the angle of propagation of the transmitted wave for the case of a plane wave incident on a boundary between air and glass with an index of refraction of 1.5.

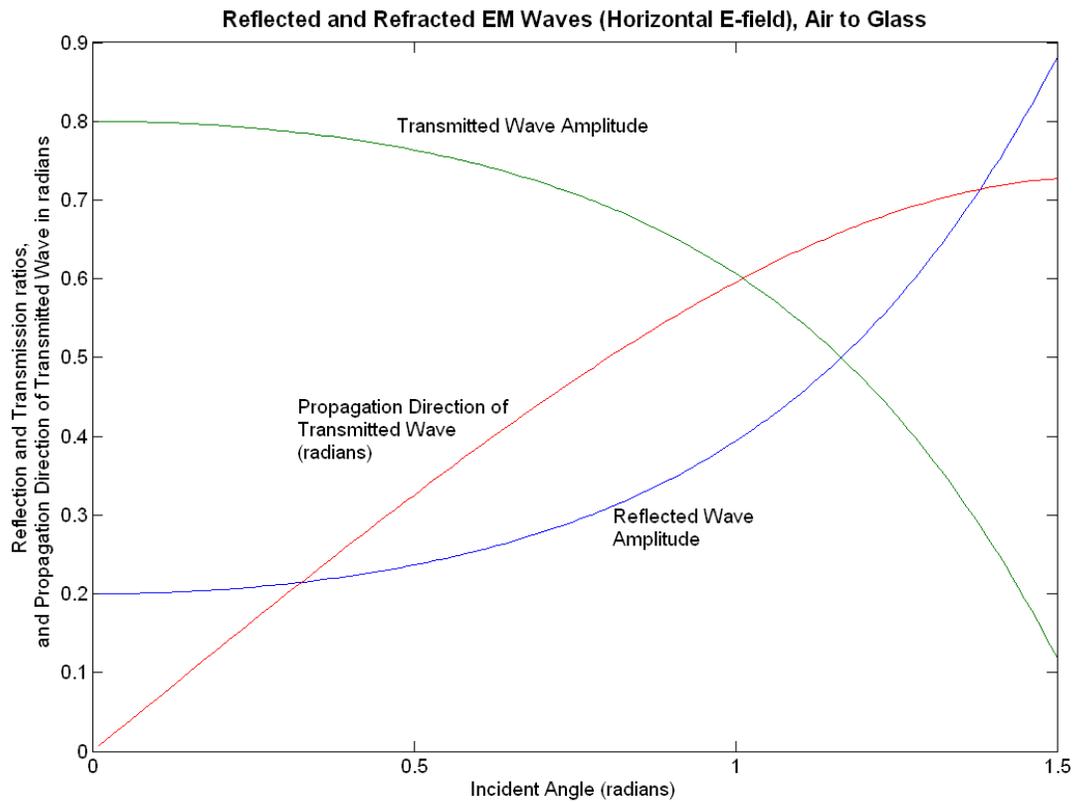**Reflected and Refracted EM Waves (Horizontal E-field), Air to Glass**

Figure 2

Figure 3 shows results for the same interface but with the E field polarized in the x-y plane. Notice that for this case there is an angle of incidence for which there is no reflected wave. This angle is called Brewster's angle.
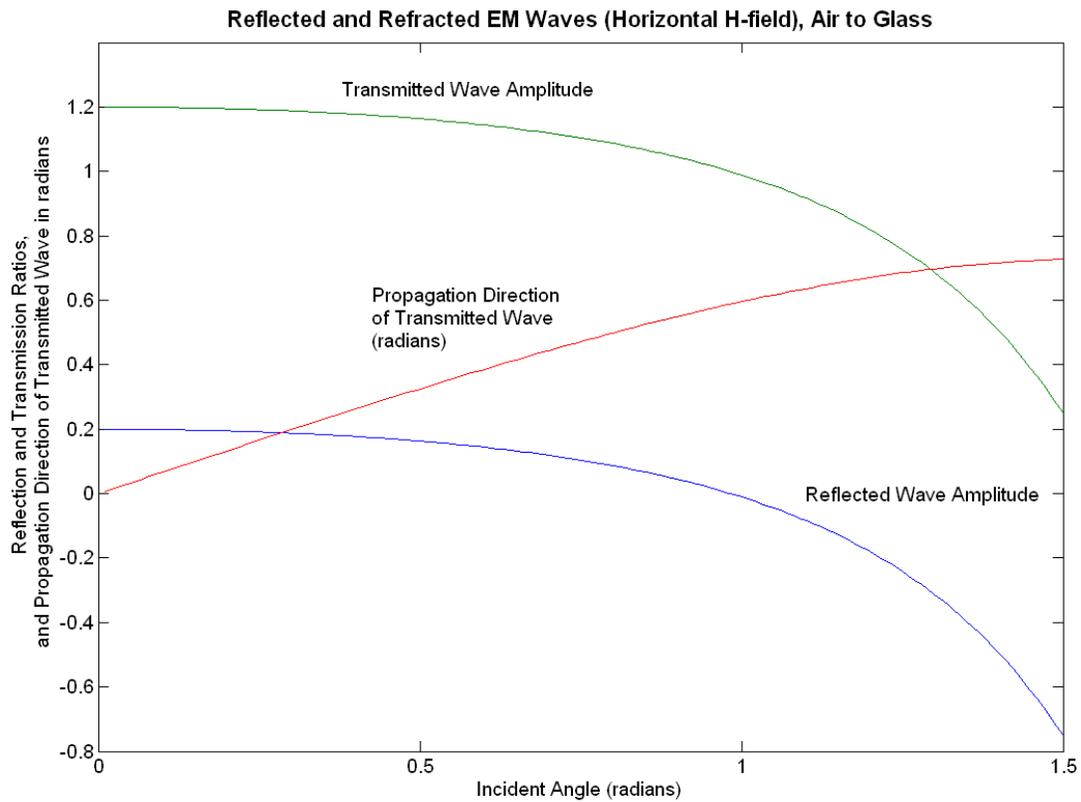
Reflected and Refracted EM Waves (Horizontal H-field), Air to Glass

Figure 3.

Acoustical Scattering at an Interface

Axes for the analogous problem in acoustics are shown in figure 4.[2]
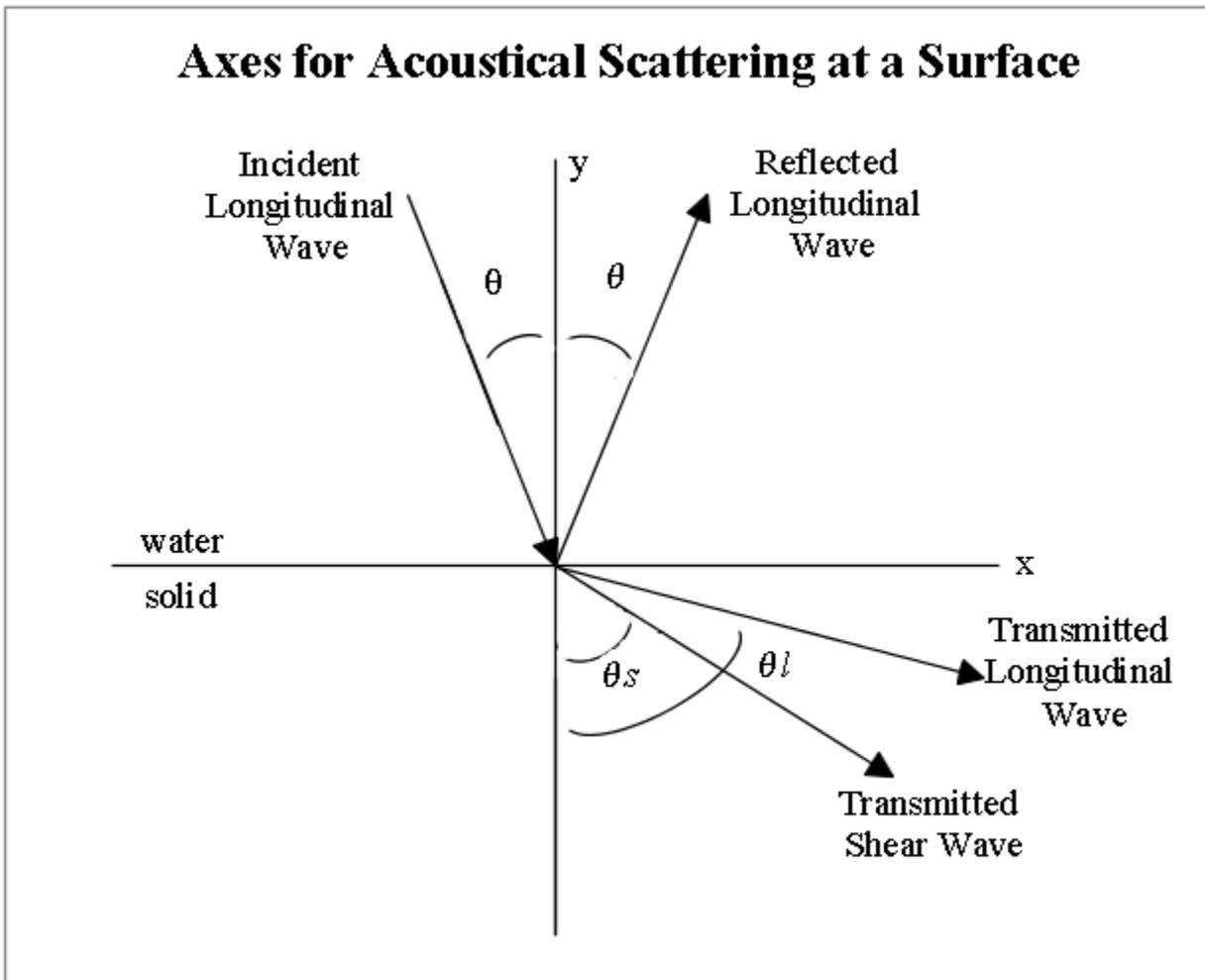
Axes for Acoustical Scattering at a Surface

Figure 4.

This is the special case of a longitudinal, or compressional, plane wave incident on a boundary between a liquid and an isotropic solid. In general, for sound waves there are three polarizations, longitudinal and two polarizations of shear; and hence, there could be two reflected waves in addition to the two transmitted waves shown. In this case however, liquid does not support shear waves and so there is only a longitudinal reflected wave.

The boundary conditions for acoustics in solids are that particle velocity and traction force must be continuous. This can be written concisely as $\mathbf{v} = \mathbf{v}'$ at $y = 0$, and $\mathbf{T} \bullet \mathbf{n} = \mathbf{T}' \bullet \mathbf{n}$ at $y = 0$, $\mathbf{v}$'s are particle velocity, $\mathbf{T}$'s are stress tensors, and $\mathbf{n}$'s are unit normal vectors. For the case shown, that of a liquid and a solid with a longitudinal incident wave, the particle velocity condition is just $v_y = v_y'$ at $y = 0$, and the traction force conditions are $T_{yy} = T_{yy}'$ at $y = 0$ and $T_{xy} = T_{xy}'$ at $y = 0$.

Proceeding as in the EM case, we write plane wave solutions for the four plane waves involved and apply the boundary conditions. The boundary condition equations are as follows, written in matrix form.

$$\begin{pmatrix} \cos\theta & -\sin\theta_s & \cos\theta_l \\ \dfrac{-C_{11w}}{\sin\theta} & -2C_{44s}\cos\theta_s & \dfrac{C_{11s}}{\sin\theta_l} - 2C_{44s}\sin\theta_l \\ 0 & -\sin\theta_s + \dfrac{\cos^2\theta_s}{\sin\theta_s} & 2\cos\theta_l \end{pmatrix} \begin{pmatrix} R \\ T_s \\ T_l \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \dfrac{C_{11w}}{\sin\theta} \\ 0 \end{pmatrix} A \qquad (1)$$

Variables are, θ = incident angle = reflected angle, θ_s = transmitted angle of shear wave, θ_l = transmitted angle of longitudinal wave, C_{11w} =compressional elastic constant of water, C_{11s} = compressional elastic constant of the sample, C_{44s} =shear elastic constant of the sample. A, R, T_s, and T_l are the amplitudes of the incident, reflected, transmitted shear, and transmitted longitudinal waves respectively.

Figure 5 shows the angles of the two transmitted waves and the amplitudes of transmitted and reflected waves for the case of water and aluminum.
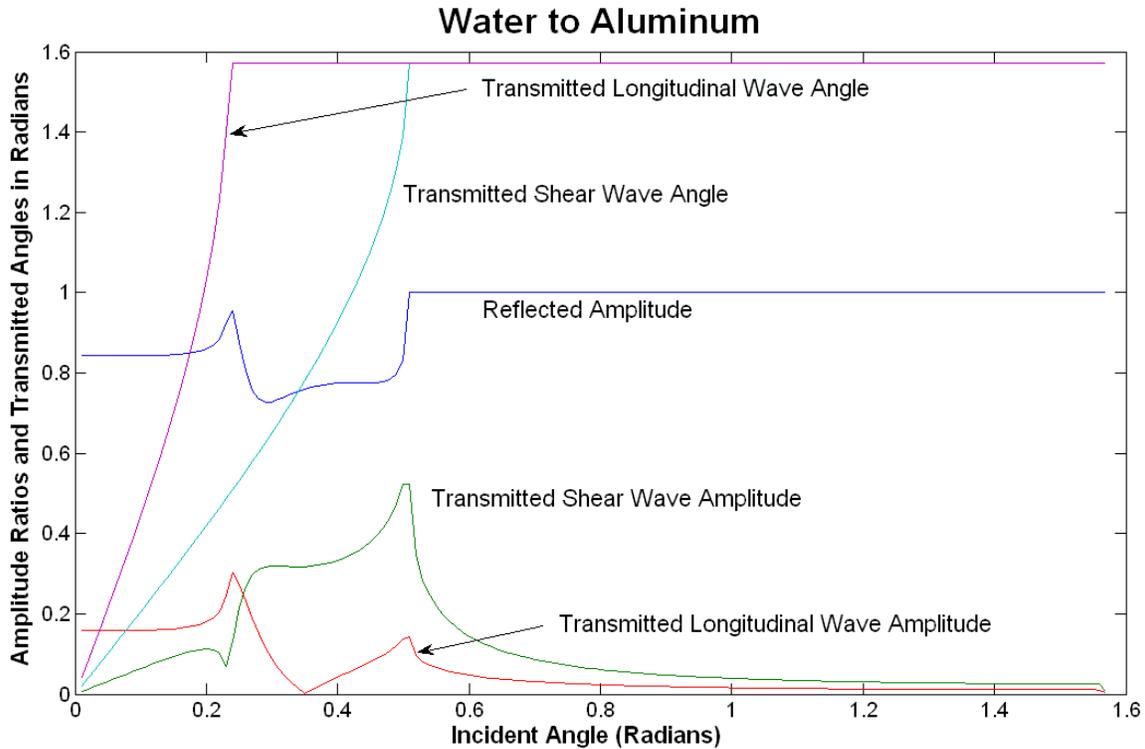

Water to Aluminum

Figure 5.

For angles less than 0.23 radians, both shear and longitudinal waves are transmitted. At about 0.24 radians, the transmitted angle of the longitudinal wave goes to 1.57 radians, or 90 degrees. It is now an evanescent wave on the surface and not carrying any energy

normal to the surface. At about 0.52 radians, the same thing happens to the shear wave, and at angles higher than that, the reflected wave amplitude is 1; in other words, the energy is completely reflected, or there is "total internal reflection", internal to the water. Notice that in this case, below 0.4 radians, the amplitude of both transmitted waves is small compared to the reflected wave. This is the opposite of the EM case, and means that the measured received signal is going to be small. Fortunately, they can be easily measured with the oscilloscope.

Refraction at a 90 degree corner

To calculate the included angle and the amplitude ratios, a similar problem needs to be solved at the second interface. This problem will be a longitudinal (shear) wave in the solid incident on water. The setup for it is another 3x3 matrix equation with solutions similar to those in figure 5; and the overall amplitude ratio, from input to output, is the product of the longitudinal (shear) amplitude curve in figure 5 and a similar curve from the solution to the second boundary problem. The matrix equations for these two cases, and the examples of their solutions, are given in Appendix 1. A Matlab program is given for calculating the overall transmission ratio.

Results of this analysis for an aluminum sample in water are given in the following table.

Scattering Coefficients for Aluminum Sample in Water (Calculated)			
Values given correspond to a perfectly symmetrical arrangement in which the line that bisects the 90 degree corner on the aluminum sample also bisects the angle between source and receiver, and the received signal has been maximized.			
Incident Angle 9.4 degrees	Longitudinal in Water to Longitudinal in Aluminum 0.166	Longitudinal in Aluminum to Longitudinal in Water 1.38	Total 0.23
Incident Angle 20.1 degrees	Longitudinal in Water to Shear in Aluminum 0.32	Shear in Aluminum to Longitudinal in Water 1.33	Total 0.43

Experimental Measurements

The apparatus for this experiment operates in a manner similar to a prism spectrometer. (See figure 6.)
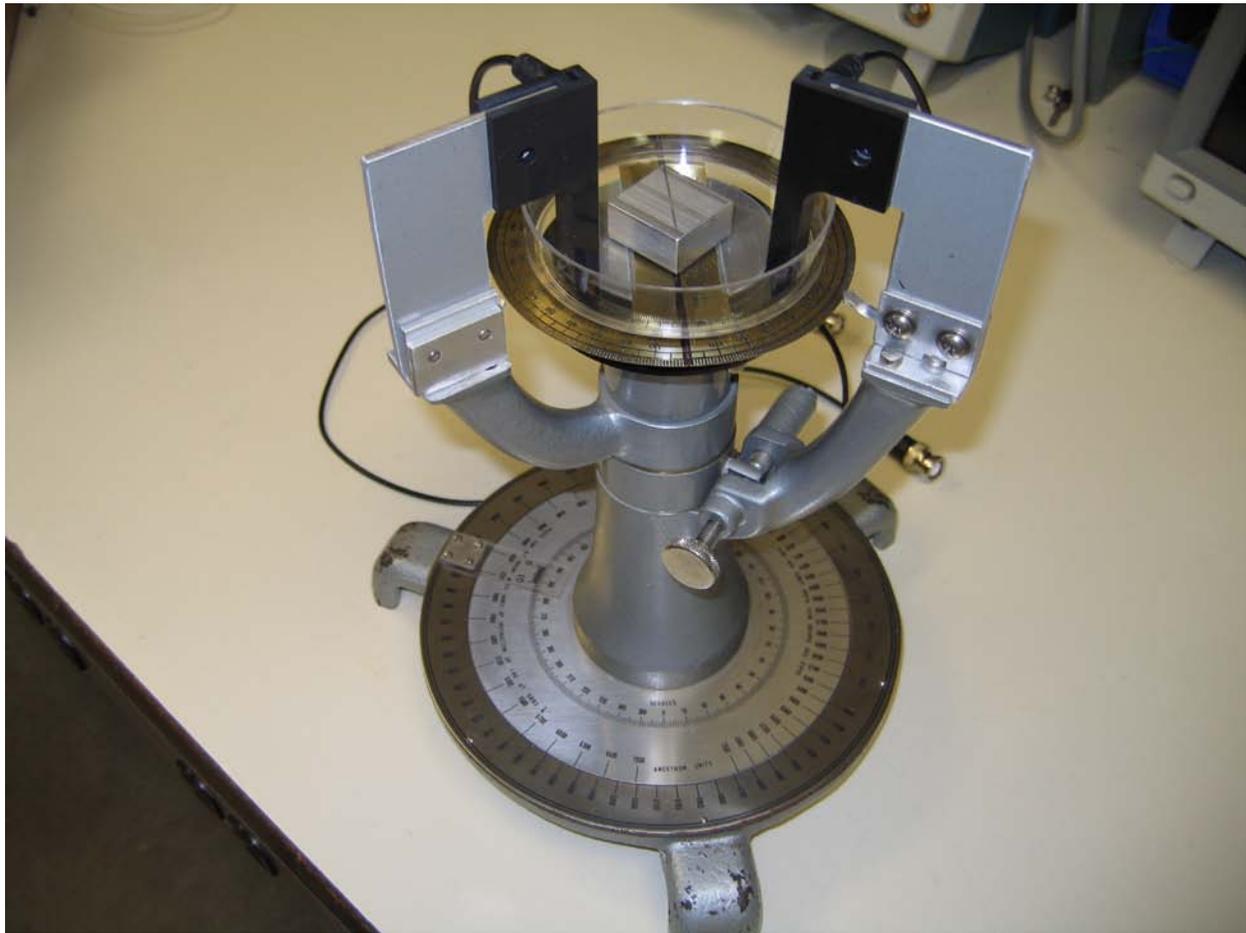

Figure 6.

There is a source of acoustic waves, a container of water with a solid sample in it, and an acoustical receiver. When the incident wave is close to the correct angle with respect to the near surface of the sample, the receiver angle can be adjusted for a maximum to find the total angle of deflection. The acoustic fields launched by the source will not be plane waves, but the analysis of the previous sections applies to any traveling wave solution form of the wave equation, and therefore applies to this case as well.

The mathematics is simplest if, when the received signal is maximized, the wedge angle of the solid is symmetric with respect to the included angle between the source and receiver. See the drawing in figure 7.
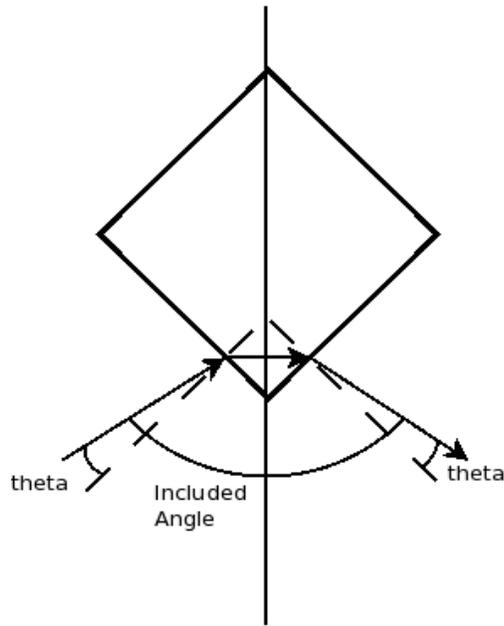
Figure 7

In that case, the included angle is exactly twice the input angle plus 180 minus the wedge angle. The graph in figure 8 shows how the included angle varies with input angle as you move away from the symmetric condition.

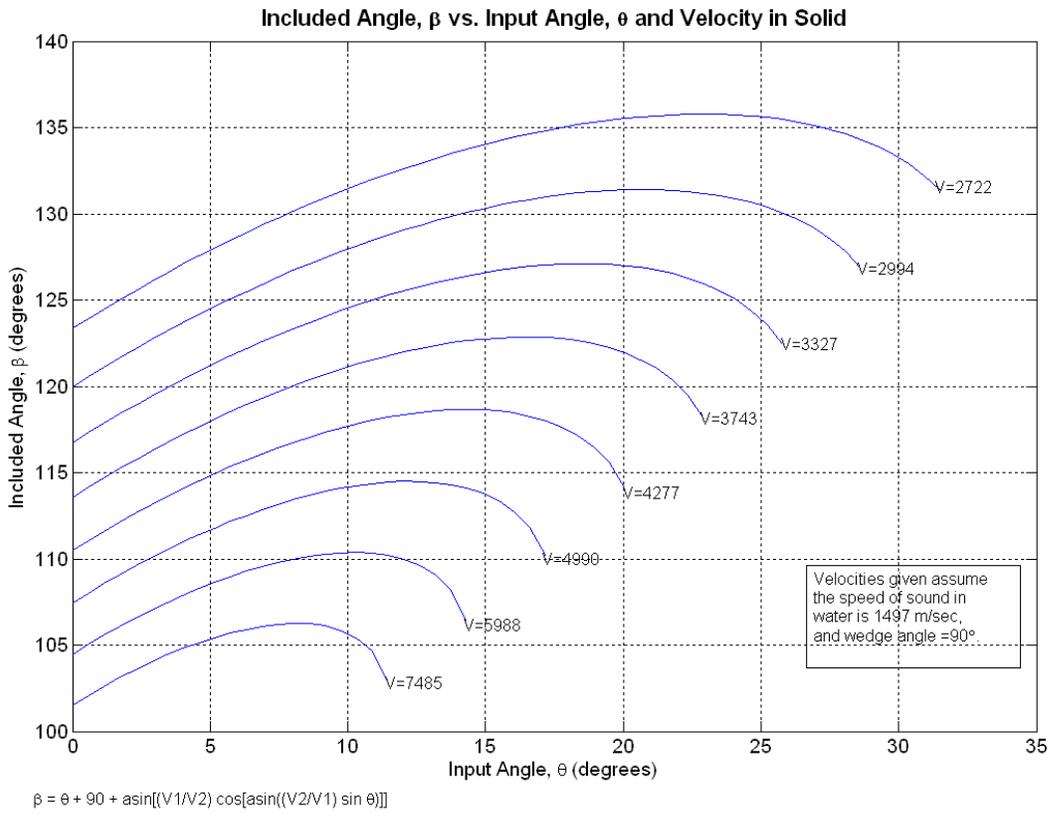$$\beta = \theta + 90 + asin[(V1/V2)\cos[asin((V2/V1)\sin\theta)]]$$

Figure 8.

The curves have maxima at the symmetry condition described above, so errors in the Input Angle will only cause small errors in the Included Angle.

This graph (or equation) can be used to estimate the error due to displacement from perfect symmetry. It also gives you a starting angular position for the block if you know approximately what the velocity is. Once you find a maximum, you can further refine the angles. From the included angles, you can calculate θ, θ_1, and θ_s. If the signal is maximized, the internal angle, θ_1 or θ_s, will be 45 degrees, and θ is the incident angle.

Once the angles for the three waves have been calculated from the measured angles, the speeds of the waves can be calculated from Snell's law, $\sin\theta / V_w = \sin\theta_1 / V_1 = \sin\theta_s / V_s$, where V_w, V_1, and V_s are sound speeds; and the elastic stiffness constants can be calculated from the formulae for the speeds, $V_w = \sqrt{\dfrac{C_{11w}}{\rho_w}}$,

$V_1 = \sqrt{\dfrac{C_{11s}}{\rho_s}}$, and $V_s = \sqrt{\dfrac{C_{44s}}{\rho_s}}$. The symbol ρ_w is the density of water, and ρ_s is the density of the sample. These formulae are for an isotopic solid and there are only two independent elastic constants. The density of water is 1000 kg/m^3. The density of the sample can be measured or copied from tables.

Figure 9 shows the speed of sound in the solid as a function of the measured included angle for several different wedge angles. The equation used to plot this graph is used to calculate the speed of sound. This graph also assumes perfect symmetry of course. From these curves, you may want to choose a wedge angle different from 90 degrees for some samples. For example, a sample with a very low acoustic velocity, a low wedge angle would give you better resolution.

Speed of Sound vs. Included Angle for Refraction Analyzer

Wedge Angle = 120° Wedge Angle = 90° Wedge Angle = 60° Wedge Angle = 30°

These curves are based on the speed of sound in water, V1 = 1497 m/sec, and that the alignment is perfectly symmetrical.

Sound speed in solid, V2 (m/sec)

Included angle, β

V2=V1[sin(α/2)/sin((β-180+α)/2)], where α is the wedge angle, β is the included angle, and V1 is the speed of sound in water.
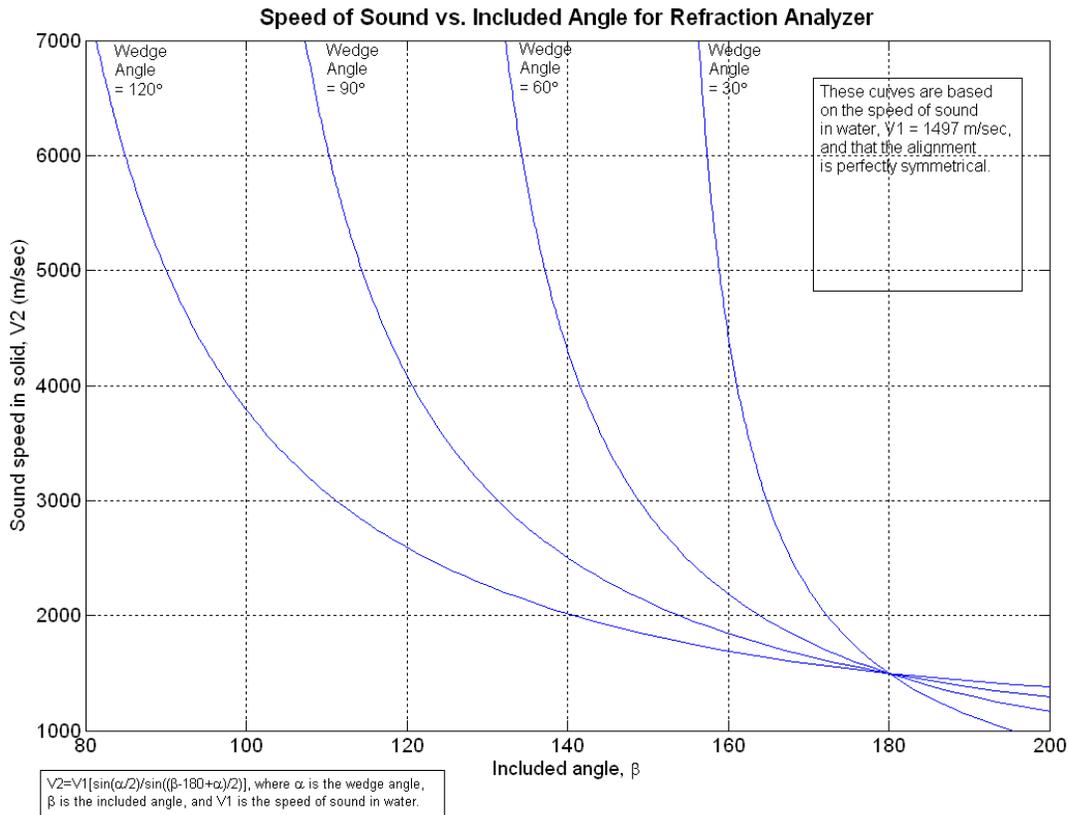
Figure 9.

This procedure can be used to determine both the longitudinal and the shear speeds of sound in the solid. From these, and the density of the sample, all the elastic constants of the sample can be calculated.

Time-of-Flight Methods of Determining Acoustic Velocities

The result for the longitudinal wave can be compared to another independent method of measuring acoustic speed, the time of flight method. This method simply uses the difference in time of flight with and without a sample between source and receiver to give a velocity. [3] The expression is $V_s = \dfrac{V_w}{1 - V_w \dfrac{\Delta t}{L_s}}$, where L_s is the sample length.

The result of the shear wave also can be compared to an independent measurement. It is done as follows. Once the sample, source, and receiver have been positioned for the included angle measurement, the sample can be moved along the center line of the sample while observing the change in time delay from input to output. The change in time is due to the fact that the sound is spending more (less) time in aluminum and less (more) time in water, depending on which way you move it. The change in time is given by

$$\Box t = \frac{d\sqrt{2}}{\cos\theta}\left(\frac{\sin(45+\theta)}{V_s} - \frac{1}{V_w}\right),$$

where d is the distance the sample moved, θ is the incident angle, V_s is the sound velocity in the sample, and V_w is the sound velocity in water.

Amplitude Measurements

Once the velocities, and hence the elastic constants, have been measured, the amplitude measurements can be made and used as another independent check on the results. Record the amplitudes measured at the location of maximum signal strength. This is to be compared to the calculated values from the programs given in the appendix. One example of a calculated output signal is given in the table on page 8. For typical surfaces, only the ratio of the two amplitudes will be comparable to the calculated values. Rough surfaces cause a lot of loss at the interface which is not taken into account in the calculations in the table.

An Acoustical Diffraction Grating

An acoustical diffraction grating is available, see figure 10, with which the acoustical diffraction pattern can be measured. The grating was machined with a mill with a digital readout, and the pitch of the lines is very precise. For this experiment, the source transducer should be excited with a sine wave burst in the range of 9 MHz to 16 MHz. Outside of that range, the transducer response is very low and the measurements are noisy. Since both transducers are aimed at the center point of the pedestal, the grating should be placed there also. Measurements of the diffracted amplitude can then be made at various angles and various frequencies. To compare amplitudes at different frequencies, the response of the transducers should be measured by measuring the amplitude versus frequency with the two facing each other on opposite sides of the tray. The theoretical response of the grating is the same as that for an optical grating, which can be found in physics text books, or lab manuals[5].
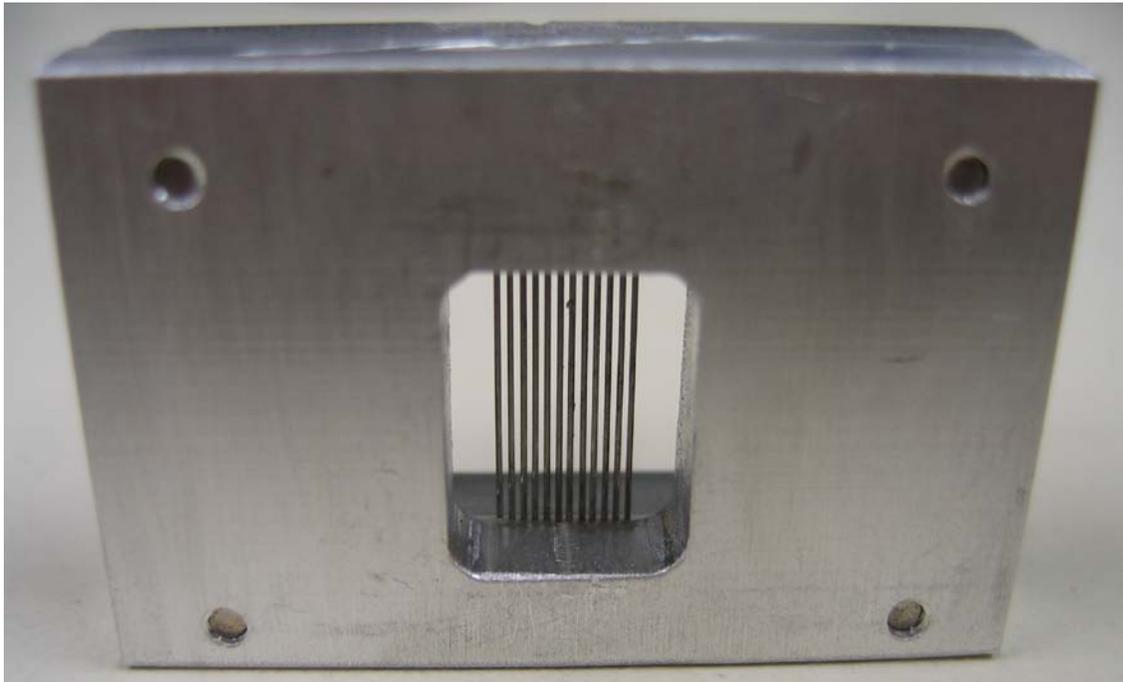
Figure 10. Acoustical Diffraction Grating

Single Slit Diffraction

The other diffraction experiment is acoustical single slit diffraction. A slit in a solid sheet is placed at the center point of the pedestal, and the source transducer is excited with a burst as before. The receiver transducer is used to make amplitude measurements at various angles and various frequencies. The theoretical response of the slit is similar to the optical slit and found in physics text books. However there is a difference. A single slit for optics is made of either metal or photographic film, and no light passes through the material outside of the slit. For acoustics this is not the case. An accurate model of a single slit in metal must include the transmission through the metal as described in the next section. As will be seen there, the transmission through the metal may vary greatly depending on the thickness of the metal.

Transmission Through Thin Plates

Transmission of acoustic waves through thin plates can be measured with the refractometer. Three different thicknesses of plates are provided, representing 1 to 2 wavelengths in the material. The transducers are positioned opposite each other and amplitude measurements are made at various angles of the sample relative to the line between transmitter and receiver. The resulting curve will be greatly different from the similar curve for transmission of an EM wave. The analysis for the thin plate is given in Appendix 3.

Acoustic Lenses

Acoustical lenses can be made out of almost any material. For solids like metals and glasses, the propagation velocity is greater than that in water; and hence, convex and concave lenses have the opposite effect as the glass lenses used in air for light. For these materials in water, the acoustical impedance mismatch is greater that glass in air for optics, so there is more reflection. However, there are materials with impedances close to that of water. RTV615 has an acoustical impedance of 1.1 MRayls compared to 1.5 MRayls for water, a propagation velocity of 1 km/sec compared to 1.5 km/sec for water, and it is fairly low loss.[4] Lenses made from this material focus acoustics in water very much like they focus light in air.

Using an acoustic lens, the beam from the source transducer can be focused to a point or spread out to a large area. The width of the beam can be measured with the receiver transducer. The focal lengths of the lenses can be measured by some of the same methods used for optical lenses.

Bibliography

1. B. A. Auld, Acoustic Fields and Waves in Solids, Vol. I, Chapter 4, John Wiley and Sons, 1973

2. Reference 1, Vol. II, Chapter 9

3. Experiments in Mechanics, Wave Motion and Heat, Laboratory Manual for 29:011, 29:027, & 29:081, Dept. of Phys. & Astron., The University of Iowa, Experiment W1: The Speed of Sound

4. A. R. Selfridge, Approximate Material Properties in Isotropic Materials, IEEE Transactions on Sonics and Ultrasonics, Vol. SU-32, No. 3, May 1985

5. Experiments in Electrics, Optics, and Modern Physics, Laboratory Manual for 29:012, 29:028, & 29:082, Dept. of Phys. & Astron., The University of Iowa, Experiment OP5: Diffraction

Appendix 1. Sample to Water Scattering

The matrix equation for an incident longitudinal wave in a solid on water is as follows.

$$
\begin{pmatrix}
\cos\theta_l & -\sin\theta_s & \cos\theta \\
-\dfrac{C_{11s}}{\sin\theta_l} + 2C_{44s}\sin\theta_l & 2C_{44s}\cos\theta_s & \dfrac{C_{11w}}{\sin\theta} \\
-2\cos\theta_l & \sin\theta_s - \dfrac{\cos^2\theta_s}{\sin\theta_s} & 0
\end{pmatrix}
\begin{pmatrix}
R_l \\
R_s \\
T
\end{pmatrix}
=
\begin{pmatrix}
\cos\theta_l \\
\dfrac{C_{11s}}{\sin\theta_l} - 2C_{44s}\sin\theta_l \\
-2\cos\theta_l
\end{pmatrix}
A_l
$$

The symbols are the same as in equations 1 except the amplitudes R_l, R_s, T, and A_l are reflected longitudinal, reflected shear, transmitted longitudinal, and incident longitudinal respectively.

The resulting reflected and transmitted amplitudes for aluminum on water are shown in figure A1-1. Notice in this case there are no discontinuities. Since the longitudinal wave velocity in aluminum is greater than any of the other waves, all the waves exist for all angles.
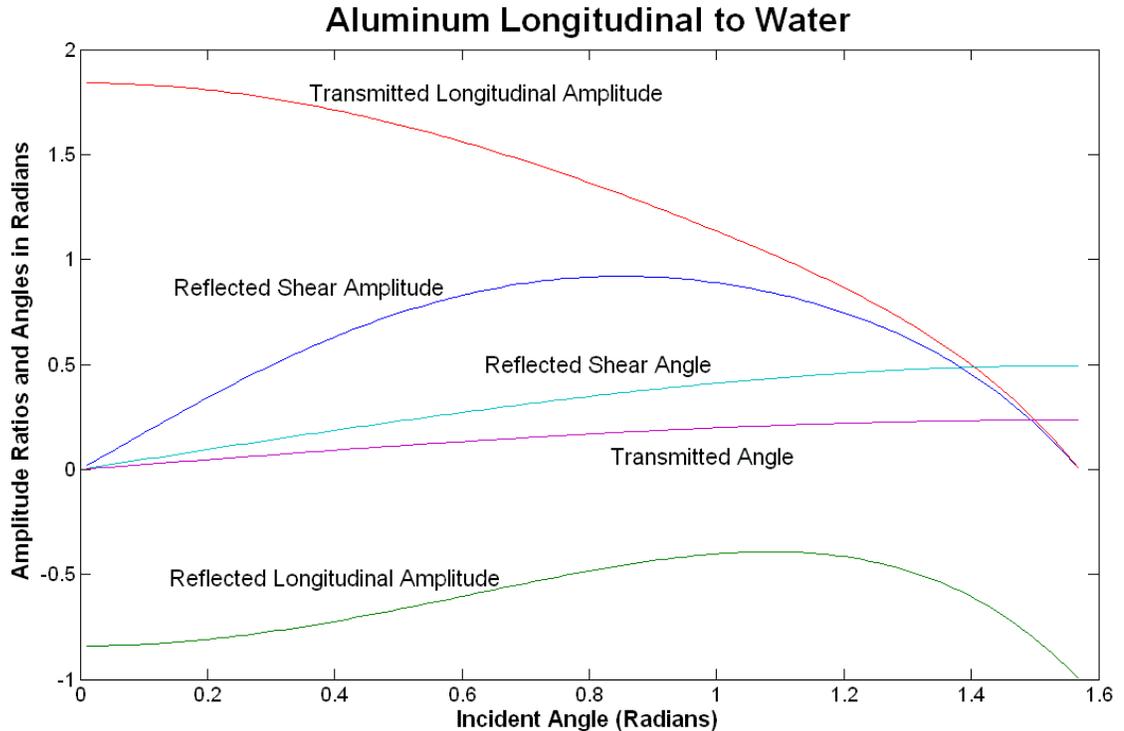
Figure A1-1.

The matrix equation for an incident shear wave in the sample on water is as follows.

$$
\begin{pmatrix}
-\sin\theta_s & -\cos\theta_l & -\cos\theta \\
2C_{44s}\cos\theta_s & \dfrac{C_{11s}}{\sin\theta_l}-2C_{44s}\sin\theta_l & -\dfrac{C_{11w}}{\sin\theta} \\
-\dfrac{\cos^2\theta_s}{\sin\theta_s}+\sin\theta_s & 2\cos\theta_l & 0
\end{pmatrix}
\begin{pmatrix} R_s \\ R_1 \\ T \end{pmatrix}
=
\begin{pmatrix}
\sin\theta_s \\
2C_{44s}\cos\theta_s \\
\dfrac{\cos^2\theta_s}{\sin\theta_s}-\sin\theta_s
\end{pmatrix} As
$$

The resulting reflected and transmitted amplitudes are shown in figure A1-2. The discontinuity in these curves is at around 0.5 radians, where the reflected longitudinal wave no longer exists. Note that at this point, the transmitted wave is nearly zero, and then at larger angles it becomes large again. This is like a Brewster's angle in reverse. At a certain angle the incident wave is totally reflected and nothing is transmitted, but it is partially transmitted at all other angles.
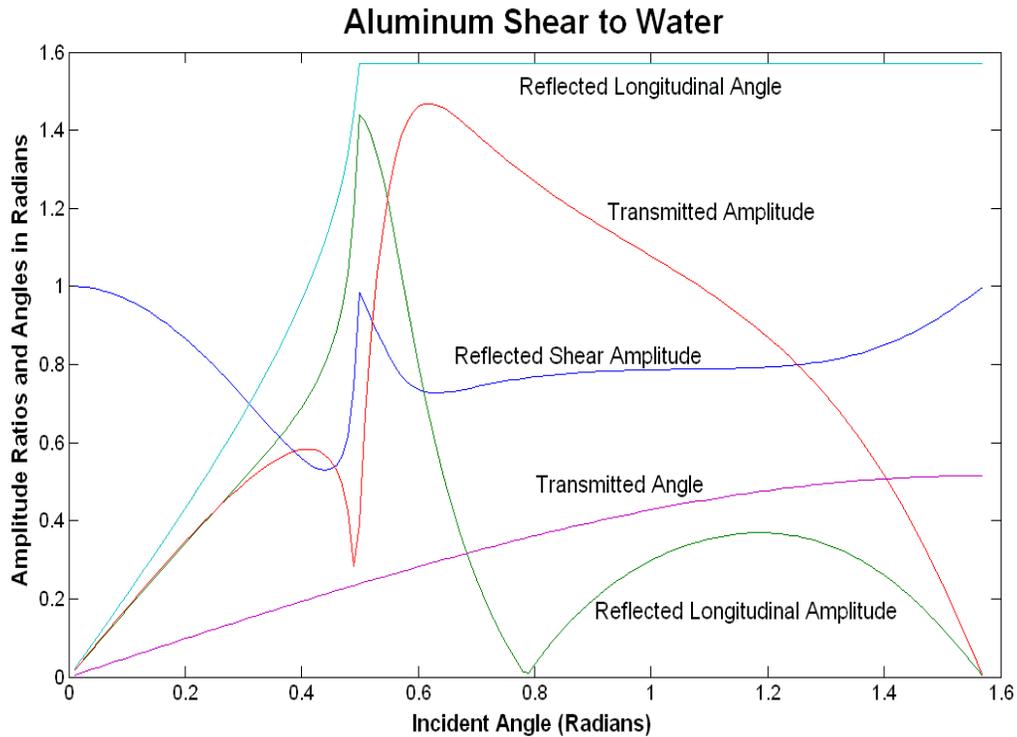

Figure A1-2.

Appendix 2. Water to Sample to Water Amplitude

To calculate the overall scattering coefficient for the case of a 90 degree corner in water with a longitudinal wave in the sample, use the program shown below. This was written in Matlab, but could be programmed in any mathematical language. The only changes would be in the statements for the matrix manipulation. As stated previously, this program assumes perfect alignment of the angles. The input data for this program are the values for the five parameters given at the top of the program. Put your values in for rhowater, rhosample, C11water, C11sample, and C44sample.

```
%WatertoLongitudinaltoWaterAmplitude

% A program for calculating the acoustical refraction amplitude for 45 degrees
% in sample.
% This is for a longitudinal wave in water to a longitudinal wave in the
% sample to a longitudinal wave in water.
% rhowater is the density of water, rhosample is the density of the sample.
% C11water is the elastic constant of water.
% C11sample and C44sample are the elastic constants of the sample.

rhowater=1e3; rhosample=2.7e3;
C11water=2.2e9; C11sample=11.1e10; C44sample=2.5e10;

Vw=(C11water/rhowater)^.5; Vl=(C11sample/rhosample)^.5;
Vs=(C44sample/rhosample)^.5; r=rhosample/rhowater;
thetal=pi/4; Sl=sin(thetal); Cl=cos(thetal);
S=(Vw/Vl)*Sl; C=(1-S^2)^.5;
Ss=(Vs/Vl)*Sl; Cs=(1-Ss^2)^.5;
theta=asin(S);

M=[C, -Ss, Cl;
     -Vw^2/S, -2*Vs^2*r*Cs, (Vl^2/Sl-2*Vs^2*Sl)*r;
     0, -Ss+Cs^2/Ss, 2*Cl];
D=[C, Vw^2/S, 0]';
B=M\D;
B(3);

M=[-Ss, Cl, C;
     2*r*(Ss/S)^2*Cs, -r*(Sl/S^2)+2*r*(Ss/S)^2*Sl, 1/S;
     Ss-Cs^2/Ss, -2*Cl, 0];
D=[Cl, r*(Sl/S^2)*(1-2*Ss^2), -2*Cl]';
F=M\D;
TotalWaterLongitudinalWater=B(3)*F(3)
```

To calculate the overall scattering coefficient for the case of a 90 degree corner in water with a shear wave in the sample, use the following Matlab program. This program works the same way as the previous one. You need to put in your values for the five parameters. There are no matrix statements in this one.

```
%WatertoSheartoWaterAmplitude
```

```
% A program for calculating the acoustical refraction amplitude.
% This is for a longitudinal wave in water into a shear wave in
% aluminum into a longitudinal wave in water. The angle in the
% aluminum is 45 degrees.
% rhowater is the density of water, rhosample is the density of the sample.
% C11water is the elastic constant of water,
% C11sample and C44sample are the elastic constants of the sample.

rhowater=1e3; rhosample=2.7e3;
C11water=2.2e9; C11sample=11.1e10; C44sample=2.5e10;

Vl=(C11sample/rhosample)^.5; Vw=(C11water/rhowater)^.5;
Vs=(C44sample/rhosample)^.5; r=rhowater/rhosample;
thetas=pi/4; theta=asin((Vw/Vs)*sin(thetas));
N1=-2*cos(theta)*r*Vw^2/sin(theta);
D1=2*Vs^2*cos(thetas)*cos(theta)+r*Vw^2*(sin(thetas)/sin(theta));
N2=-4*Vs^2*cos(thetas)*sin(thetas);
D2=2*Vs^2*cos(thetas)*cos(theta)+r*Vw^2*(sin(thetas)/sin(theta));
TotalWaterShearWater=N1*N2/(D1*D2)
```

Appendix 3. Thin Plates

The analysis for the thin plate is similar to the single interfaces given earlier, but now there are two interfaces and six boundary conditions to match, resulting in a 6 by 6 matrix equation. Figure A3-1 shows the arrangement for the analysis.
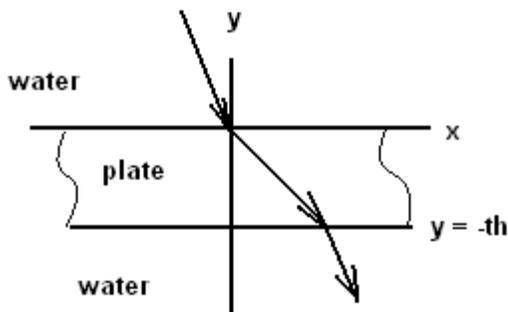

Figure A3-1

For the matrix equation, the following variables are defined:

θ = incident angle

θ_s = angle of shear wave in plate

θ_l = angle of longitudinal wave in plate

C11w = compressional elastic constant of water

C11s = compressional elastic constant of the sample

C44s = shear elastic constant of the sample

Vw = longitudinal velocity in water

Vs = shear velocity in the sample

Vl = longitudinal velocity in the sample

th = plate thickness

$$el = e^{i2\pi th \cos\theta_l \frac{V_w}{V_l}}$$

$$es = e^{i2\pi th \cos\theta_s \frac{V_w}{V_s}}$$

$$et = e^{i2\pi th \cos\theta}$$

The matrix can now be written as follows:

M = [cosθ, -sinθ$_s$, cosθ$_l$, -sinθ$_s$, -cosθ$_l$, 0;

-Cllw/sinθ, -2*C44s*cosθ$_s$, C11s/sinθ$_l$-2*C44s*sinθ$_l$, 2*C44s*cosθ$_s$, C11s/sinθ$_l$-2*C44s*sinθ$_l$, 0;

0, cos^{2}θ$_s$/sinθ$_s$-sinθ$_s$, 2*cosθ$_l$, cos^{2}θ$_s$/sinθ$_s$-sinθ$_s$, -2*cosθ$_l$, 0;

0, -sinθ$_s$/es, cosθ$_l$/el, -sinθ$_s$*es, -cosθ$_l$*el, -cosθ/et;

0, -2*C44s* cosθ$_s$/es, (C11s/sinθ$_l$-2*C44s*sinθ$_l$)/el, 2*C44s* cosθ$_s$*es, (C11s/sinθ$_l$-2*C44s*sinθ$_l$)*el, -C11w/(sinθ*et);

0, (cos^{2}θ$_s$/sinθ$_s$-sinθ$_s$)/es, 2* cosθ$_l$/el, (cos^{2}θ$_s$/sinθ$_s$-sinθ$_s$)*es, -2* cosθ$_l$/el, 0]

The input vector is

[cosθ, C11w/sinθ, 0, 0, 0, 0]

The sixth element of the solution vector for this matrix equation is the amplitude of the transmitted wave, and this is what is plotted in the graphs.

Using the matrix equation to calculate the amplitude of a signal after transmission through thin plates results in the graphs of figures A3-2 through A3-4. These are graphs of amplitude versus incident angle for three different thickness of stainless steel plates, 0.05 mm, 0.1 mm, and 0.2 mm, at a frequency of 12 MHz.
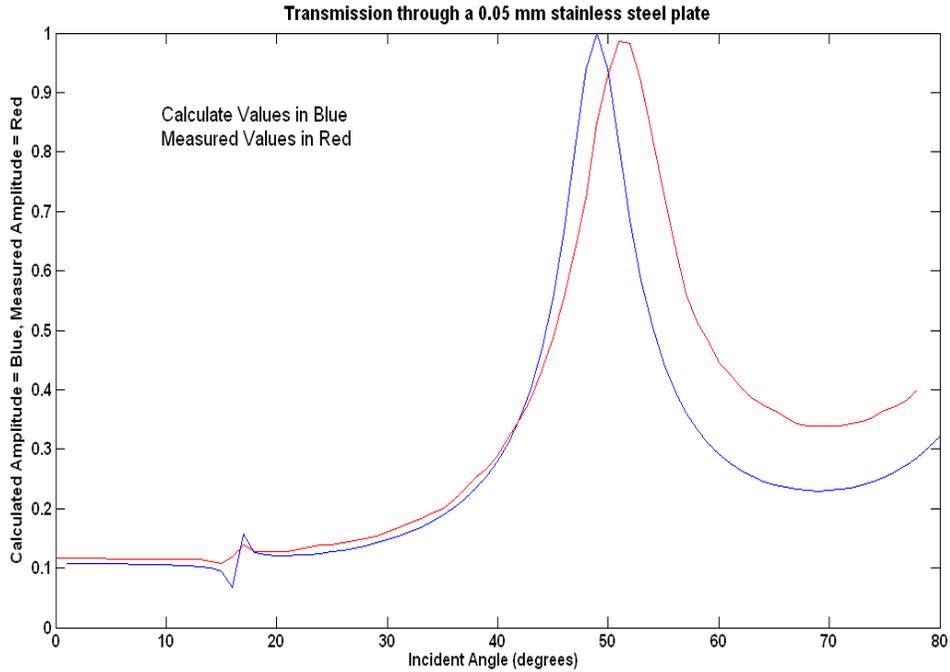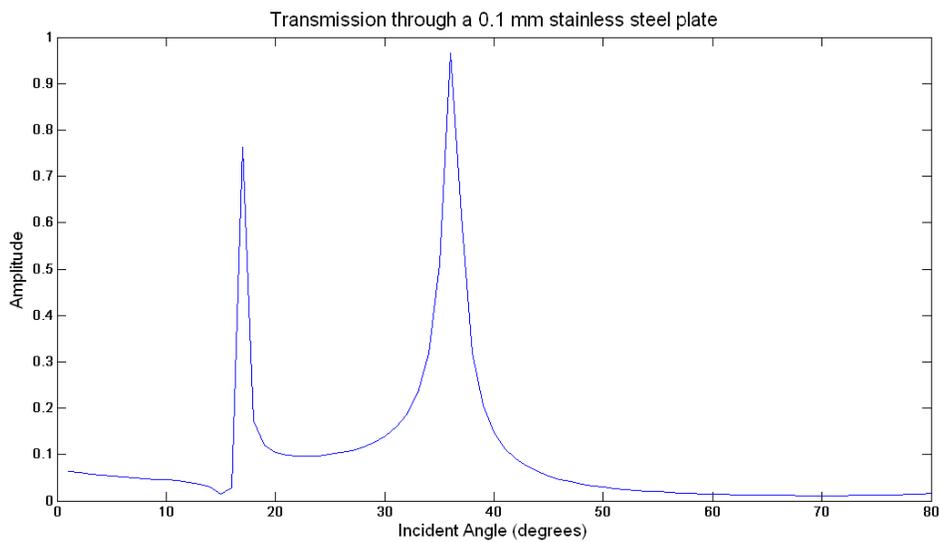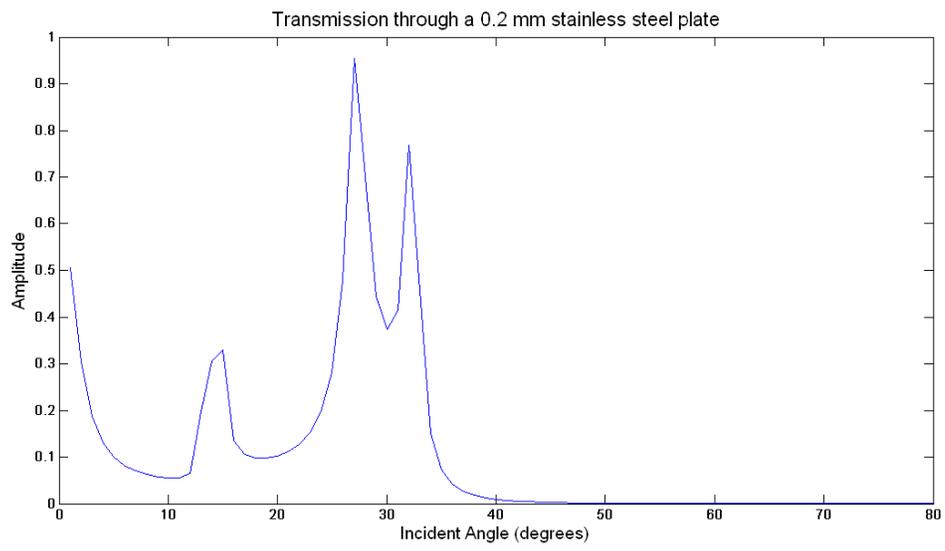

Figure A3-2

Figure A3-3

Figure A3-4