

Acoustic Velocity, Impedance, Reflection, Transmission, Attenuation, and Acoustic Etalons

Acoustic Velocity

The equation of motion in a solid is ⁽¹⁾

$$\nabla \cdot \mathbf{T} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \quad (1)$$

where \mathbf{T} is the stress tensor, ρ is the density, and \mathbf{u} is particle displacement. Consider a longitudinal wave traveling in the x direction so that there is only an x component of \mathbf{u} . Then the wave equation simplifies to

$$\frac{\partial T_{11}}{\partial x} = C_{11} \frac{\partial S_{11}}{\partial x} = C_{11} \frac{\partial^2 u_1}{\partial x^2} = \rho \frac{\partial^2 u_1}{\partial t^2} \quad (2)$$

This is a standard wave equation, which has a variety of solutions. All traveling wave solutions have a wave velocity $V_a = \sqrt{\frac{C_{11}}{\rho}}$. All acoustic waves will have a mathematical form similar to this, the square root of the ratio of an elastic constant to the density. The analogous form for EM waves is $\sqrt{\frac{1}{\epsilon\mu}}$.

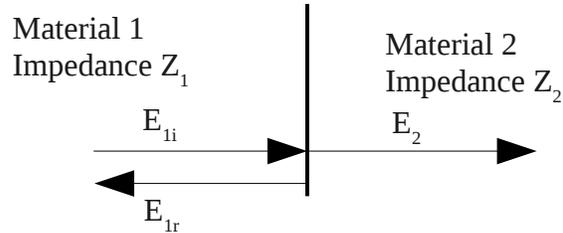
Acoustic Impedance

The intrinsic impedance for an EM wave is the magnitude of the ratio $\frac{\mathbf{E}}{\mathbf{H}}$ and, if \mathbf{E} and \mathbf{H} are those of a simple traveling wave, is equal to $\sqrt{\frac{\mu}{\epsilon}}$. In acoustics, the usual analogies ⁽¹⁾ are \mathbf{T} to \mathbf{E} and \mathbf{v} to \mathbf{H} , where \mathbf{v} is particle velocity, $\mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}$. Under this analogy, intrinsic acoustic impedance, also called stress acoustic impedance, is equal to $Z_a = \left| \frac{\mathbf{T}}{\mathbf{v}} \right| = \sqrt{\rho C}$. In the example of the last section, it would be $\sqrt{\rho C_{11}}$. Notice that this acoustic impedance is the density, ρ , times the acoustic velocity, $Z_a = \rho V_a$.

1 B. A. Auld, p106

Reflection and Transmission

Once we have a concept of intrinsic impedance, we can write formulae for reflection at an interface between two materials by analogy with the results for EM waves. The formula for EM reflection at the interface shown in the figure is



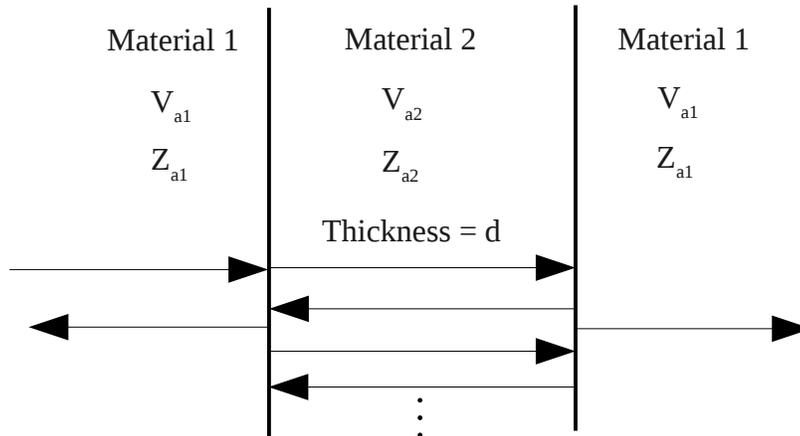
$$\left| \frac{E_{1r}}{E_{ii}} \right| = \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (2)$$

This is an E-field reflection coefficient and represents the ratio of the amplitude of the E-field of the reflected wave to that of the incident wave. The acoustic reflection coefficient will have the same form using the acoustic impedances.

The same analogies are true for the transmitted wave and the transmission coefficient can be written as $T = \frac{2Z_2}{Z_2 + Z_1}$.

Acoustic Etalon

Optical etalons or resonators are commonly used as filters and Fourier transformers for optical experiments⁽³⁾. The acoustical etalon is nearly an exact analogy using the identifications listed in Part 2. The figure below shows the arrangement.



The term “finesse” is used as an indication of how narrow the filter is. It is defined as the “free spectral range” divided by the “width” of a resonant peak. “Free spectral range” is the spectral distance between resonant peaks.

The approximate formula for the Finesse of an etalon is $Finesse = \pi\Gamma/(1-\Gamma^2)$; but for convenience, we will use an on-line etalon calculator⁽⁴⁾ to show how different etalon parameters affect the finesse of the device, and to show the limits of an acoustic etalon. The five parameters that dominate the properties of an acoustical etalon are: thickness, surface reflectivity, surface smoothness, acoustic attenuation, and surface parallelism. The calculator will show the limit of finesse due to each of these and the total finesse, so that each can be modified to observe the effect on the total. The calculator is at reference 5. The graphical part of this interactive calculator will only work with Internet Explorer.

If there is no attenuation, the reflection coefficient at the boundary of the etalon can be used as the surface reflectivity in the on-line workspace. This will be the reflection coefficient from the previous section,

$$\Gamma = \frac{Z_{1a} - Z_{2a}}{Z_{1a} + Z_{2a}} \quad \text{for this case.}$$

If there is attenuation, treated in the next section, the round trip loss is $1 - \Gamma^2 e^{-2\alpha d}$, and therefore the effect of the attenuation on the etalon can be lumped with the reflection coefficient. The effective reflection coefficient to use in the on-line calculator is $\Gamma e^{-\alpha d}$. Alpha, α , is the attenuation coefficient and d is thickness in these expressions.

3 www.precisionphotonics.com/technology/Etalonintroduction.pdf

4 www.lightmachinery.com/etalon-designer-r8.php

Attenuation

All materials have some acoustical attenuation, unlike optics, where some materials have almost no attenuation. The acoustical attenuation, or loss, in solids is due to thermodynamics, phonon equilibrium, lattice imperfections, and, in conductors, interactions with electrons. In liquids, there are other loss mechanisms such as microscopic bubbles and suspended solids. Acoustic attenuation can be measured in several ways. Once the acoustic impedances have been evaluated, the attenuation can be measured with the speed-of-sound setup. For a solid sample in water, the reflection coefficient from the previous section can be used. The ratio of the second emergent pulse to the first should be Γ^2 if there is no attenuation. With attenuation, the ratio is $\Gamma^2 e^{-2\alpha d}$, where d is the thickness of the sample and α is the attenuation factor.

Laboratory Activities

1. Use the Speed-of-Sound setup to measure the speed of sound in several different solids.
2. Measure the density of the solids, and from these measurements calculate the acoustic impedances.
3. Calculate reflection and transmission coefficients of these solids in water.
4. Measure the “round trip” loss by measuring the ratio of two successive pulses.
5. Is the loss in the bulk significant compared to the loss due to transmission? Calculate the loss in the bulk for a “round trip”. Attenuation, or loss, is commonly given in dB/cm at a given frequency.⁽⁵⁾ Estimate the center frequency of the received pulses and express your measured loss at this frequency.
6. Measure the speed of sound and acoustic impedance in sapphire. Calculate the reflection and transmission coefficient of sapphire in water. Calculate the attenuation coefficient for sapphire.
7. Use these measurements in the etalon calculator to predict the finesse and Free Spectral Range of the sapphire etalon.

This calculator is normally used for optics. In optics, reflection and transmission coefficients are usually measured in terms of intensity rather than amplitude. For an EM wave, intensity is defined as E^2/Z , where E is the rms amplitude of the wave's E-field, and Z is the intrinsic impedance of the medium. Its dimensions are power per unit area.

The reflection and transmission coefficients we measured are in terms of amplitude; and therefore, are proportional to the square root of those based on intensity. What you put into the calculator for surface reflectivity will be the square of the reflection coefficient you measured.

Likewise, to compare measured to calculated finesse, use the 3dB bandwidth of the resonant peaks instead of the full-width-at-half-maximum bandwidth used for optics.

Remember that what the calculator calls the “index” is actually a ratio of velocities inside and outside the etalon, and that is what you should put in for the “index”.

Since the calculator assumes the speed-of-light for a wave speed, you will have to multiply the FSR(GHz) and the Bandwidth(GHz) by the ratio of the speed-of-sound to the speed-of-light to get the calculated numbers to compare to your measured numbers.

8. Drive the input transducer with a burst generator with the burst frequency set to the center frequency of the received pulse estimated in part 5. Adjust the angle of the sample for maximum received signal. Set the burst length to be long enough so that the received burst reaches its maximum height. Take readings of the received amplitude at enough different frequencies to allow you to calculate the Free Spectral Range and finesse. Compare results with the calculations of part 7.

5 Onda web page.